# Analysis of Exponential Logarithmic Lifetime Distribution

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**Available online at: www.isroset.org**

*Abstract-* This paper describes the Bayesian inference and prediction of the Exponential Logarithmic distribution   . The aim of this paper is to obtain the Bayesian inference of the unknown parameters under different loss functions. The Bayes estimates can be obtained and it has been used to compute the Bayes estimates and also to construct symmetric loss function.  We consider the posterior predictive density of the future observations and also asymmetric loss function.

*Keywords*- Lifetime distributions, reliability, failure rate, order statistics, exponential distribution, truncated logarithmic distribution

## I.    INTRODUCTION

Exponential distributions are encountered as lifetime distribution with constant rate. They are commonly employed in the formation of models of lifetime distribution, stochastic process in general. The distributions with decreasing failure rate are studied in the works of Lomax (1954) Porchan (1963), Barlow et al (1963), Borlow and Marshall (1964,1965), Marshall and Porshan (1965), Cozzolino(1968),Dahiya and Garland (1972), Mcnolty et al (1980), Saunders and Myhre (1983), Nassar (1988), Gleser (1989) , Gurland and Sethurman (1994), admids and Loukas (19980 and Kus (2007).

It is remarkable that the development of appropriate Bayesian inference procedures has been quite limited. Zelllener (1997) includes an example involving Bayesian inference in the classical Pareto distribution for the special case in which scale parameter is known .Malik (1970) and earlier, Muniruzzaman (1968) were apparently the first to consider the case known scale parameter. Sinhaand Howlader (1980) studied Bayesian  estimators in the known scale case .Arnold and  Press (1983, 1986) discussed possible forms of priors and Bayesian analysis for different cases Pandey et al (1986)and Bhattacharya et al (1992) provides a recent through study of Bayesian estimation on the subject.
Despite being a Gamma family of distributions discussed by Karl person as early as 1895, it took another three and half decades for exponential distribution to appear in its own in the statistical literature .Kondo (1931) referred to the exponential distribution of standard deviation, as Pearson 's X type X distribution.

In the context of survival analysis Basu and Ebrahimi (1988,1991), Chaturvedi (1998) used this loss function for estimating the mean life time and survival analysis of the exponential model for different priors. Rojo (1987),Rai (1996),Pandey (1997) have applied the Linex loss function in various other estimation problems.

The exponential logarithmic (EL) lifetime distribution has been developed by mixing exponential distribution with logarithmic distribution and has decreasing failure rate (DFR). The present chapter considers the Bayesian analysis of EL distribution under appropriate prior distributions and derives the posterior distributions from the parameters. The Bayes estimators for different parameters and survival function have been derived under squared error loss function and LINEX loss function. In this chapter also we derived the posterior analysis of exponential logarithmic distribution under type 1 censoring and derive the survival function under squared error loss function and Linex loss function.

## II.    EXPONENTIAL LOGARITHMIC LIFETIME MODEL

Let us consider the exponential distribution with following probability distribution function (pdf)

$$f_y(y; \beta) = \beta e^{-\beta y} I_{(0, \infty)}^y \; ; \beta > 0 \qquad (2.1)$$

Further, let Z be a random variable following a logarithmic distribution with probability mass function

$$P_z(z;p) = \frac{(1-p)^z}{-Z \ln p} I_{\{1,2,3,\dots\}}^{(y)}$$

(2.2)

where 0<p<1 and $I_A^{(y)}$ is the indicator function defined as

$$I_A^{(y)} = \begin{cases} 1 & if\, y \in A \\ 0 & if\, y \notin A \end{cases}$$

We assume that the random variables Y and Z are independent, Let us define, X= $min\{Y_1, Y_2, \dots Y_n\}$.Then the conditional pdf of X given Z is

$$f_{x|z}(X|Z;\beta) = \beta Z e^{-\beta xz}$$

(2.3)

Further, the marginal pdf of X is

$$f_X(x;p,\beta) = \frac{-\beta(1-p)e^{-\beta x}}{-\ln p\,[1-(1-p)e^{-\beta x}]} I_{(0,\infty)}^{(x)}$$

(2.4)

The above distribution X is referred as the exponential logarithmic (EL) distribution.

The probability distribution function of EL distribution is monotonic decreasing with modal value $\beta(1-p)(-p \ln p)$ at x=0 for all values of parameters. The EL leads to exponential distribution with parameter β, as $p \to 1$.
The cumulative distribution function (cdf) of EL distribution is given by

$$F_X(x,p,\beta) = 1 - \frac{\ln[1-(1-p)e^{-\beta x}]}{\ln p}$$

(2.5)

Let $x_1,\dots,x_n$ be random variables from the EL distribution. We can write the pdf of X as

$$f_X(x,p,\beta) = \frac{-\beta(1-p)e^{-\beta x}}{-\ln p\,[1-(1-p)e^{-\beta x}]} I_{(0,\infty)}^{(x)}$$
$$= \frac{(1-p)\beta e^{-\beta x}\sum_{z=0}^{\infty}(1-p)e^{-\beta xz}}{(-\ln p)}$$

$$= \frac{\beta\sum_{z=0}^{\infty}(1-p)^{z+1}e^{-(z+1)\beta x}}{(-\ln p)^n}$$

Hence the likelihood function is given by

$$\prod_{i=1}^{n} f(x_i;p,\beta)$$

$$= \frac{\beta^n}{(-\ln p)^n}\prod_{i=1}^{n}\left\{\sum_{z=1}^{\infty}(1-p)^z e^{-\beta zx_i}\right\}$$

$$= \frac{\beta^n e^{-\frac{\beta}{\beta}}\sum_{z_1=1}^{\infty}\sum_{z_2=1}^{\infty}\cdots\sum_{z_n=1}^{\infty}(1-p)^{\sum_{i=1}^{n}z_i}e^{-\beta\sum_{i=1}^{n}z_ix_i}}{(-\ln p)^n}$$

(2.6)

## III. POSTERIOR ANALYSIS

**Prior Distribution:**

Let us assume that for the purpose of Bayesian analysis exponential logarithmic distribution follows Gamma distribution as a prior distribution

$$p(\beta|\bar{\beta},\alpha) \propto \beta^{\alpha-1}e^{-\frac{\beta}{\beta}}$$

(2.7)

**Theorem 1**

The posterior distribution of $\beta$, under the known prior distribution given in equation (2.7) is defined as

$$p^*(\beta|x) = \frac{\sum_{z_1=1}^{\infty}\sum_{z_2=1}^{\infty}\cdots\sum_{z_n=1}^{\infty}(1-p)^{\sum_{i=1}^{n}z_i}\beta^{n+\alpha-1}e^{-\beta\left(\sum_{i=1}^{n}z_ix_i+\frac{1}{\beta}\right)}}{\sum_{z_1=1}^{\infty}\sum_{z_2=1}^{\infty}\cdots\sum_{z_n=1}^{\infty}\left\{\frac{(1-p)^{\sum_{i=1}^{n}z_i}\Gamma(n+\alpha)}{\left(\sum_{i=1}^{n}z_ix_i+\frac{1}{\beta}\right)^{n+\alpha}}\right\}}$$

(3.1)

**Proof:**

We assume the prior distribution of β as gamma distribution with pdf given by

$$p(\beta|\bar{\beta},\alpha) \propto \beta^{\alpha-1}e^{-\frac{\beta}{\beta}}$$

(3.2)

We assume that p is known. Further we write $x = (x_1,\dots,x_n)$.

Combining the likelihood function (4.2.6) with the prior distribution (3.2) we obtain the posterior distribution of $\beta$ as

$$p^*(\beta|\mathbf{x}) = \frac{\prod_{i=1}^{n} f(x_i; p, \beta)\, p(\beta|\bar{\beta}, \alpha)}{p^*(\mathbf{x})}$$

$$= \frac{(-\ln p)^{-n} \sum_{z_1=1}^{\infty} \sum_{z_2=1}^{\infty} \cdots \sum_{z_n=1}^{\infty} (1-p)^{\sum_{i=1}^{n} z_i} \beta^{n+\alpha-1} e^{-\beta\left(\sum_{i=1}^{n} z_i x_i + \frac{1}{\bar{\beta}}\right)}}{p^*(\mathbf{x})}$$

(3.3)

where,

$$p^*(\mathbf{x}) = \int_0^{\infty} \prod_{i=1}^{n} f(x_i; p, \beta)\, p(\beta|\bar{\beta}, \alpha) d\beta$$

$$= (-\ln p)^{-n} \sum_{z_1=1}^{\infty} \sum_{z_2=1}^{\infty} \cdots \sum_{z_n=1}^{\infty} (1-p)^{\sum_{i=1}^{n} z_i}$$

$$\times \int_0^{\infty} \beta^{n+\alpha-1} e^{-\beta\left(\sum_{i=1}^{n} z_i x_i + \frac{1}{\bar{\beta}}\right)} d\beta$$

$$= (-\ln p)^{-n} \sum_{z_1=1}^{\infty} \sum_{z_2=1}^{\infty} \cdots \sum_{z_n=1}^{\infty} \left\{ \frac{(1-p)^{\sum_{i=1}^{n} z_i}\Gamma(n+\alpha)}{\left(\sum_{i=1}^{n} z_i x_i + \frac{1}{\bar{\beta}}\right)^{n+\alpha}} \right\}$$

(3.4)

Substituting the value of $p^*(\mathbf{x})$ in (3.3), the expression for the posterior distribution becomes

$$p^*(\beta|\mathbf{x})$$

$$= \frac{\sum_{z_1=1}^{\infty} \sum_{z_2=1}^{\infty} \cdots \sum_{z_n=1}^{\infty} (1-p)^{\sum_{i=1}^{n} z_i} \beta^{n+\alpha-1} e^{-\beta\left(\sum_{i=1}^{n} z_i x_i + \frac{1}{\bar{\beta}}\right)}}{\sum_{z_1=1}^{\infty} \sum_{z_2=1}^{\infty} \cdots \sum_{z_n=1}^{\infty} \left\{ \frac{(1-p)^{\sum_{i=1}^{n} z_i}\Gamma(n+\alpha)}{\left(\sum_{i=1}^{n} z_i x_i + \frac{1}{\bar{\beta}}\right)^{n+\alpha}} \right\}}$$

(3.5)

## IV. ESTIMATION OF PARAMETERS AND SURVIVAL FUNCTION UNDER SQUARED ERROR LOSS

For an estimator $\hat{\beta}$ of $\beta$ the squared error loss function (SELF) is defined as

$$L_1 = \left(\hat{\beta} - \beta\right)^2 \qquad (4.1)$$

Under the above squared error loss function, the Bayes estimator of $\beta$ is the posterior mean given by

$$\tilde{\beta} = E(\beta|\mathbf{x})$$

### Theorem2

The posterior mean of parameter $\beta$ of exponential Logarithmic distributions is defined under the prior distribution as given in equation (2.7) as

$$\tilde{\beta} = (n+\alpha) \frac{\sum_{z_1=1}^{\infty} \sum_{z_2=1}^{\infty} \cdots \sum_{z_n=1}^{\infty} \left\{ \frac{(1-p)^{\sum_{i=1}^{n} z_i}}{\left(\sum_{i=1}^{n} z_i x_i + \frac{1}{\bar{\beta}}\right)^{n+\alpha+1}} \right\}}{\sum_{z_1=1}^{\infty} \sum_{z_2=1}^{\infty} \cdots \sum_{z_n=1}^{\infty} \left\{ \frac{(1-p)^{\sum_{i=1}^{n} z_i}}{\left(\sum_{i=1}^{n} z_i x_i + \frac{1}{\bar{\beta}}\right)^{n+\alpha}} \right\}}$$

(4.2)

### Proof :

The posterior mean of $\beta$ is defined as

$$\delta^*(\mathbf{x}) = E(\beta/\mathbf{x})$$

$$= \int_0^{\infty} \beta\, p^*(\beta|\mathbf{x}) d\beta$$

(4.3)

where,

$$p^*(\beta|\mathbf{x}) = \frac{\prod_{i=1}^{n} f(x_i; p, \beta)\, p(\beta|\bar{\beta}, \alpha)}{p^*(\mathbf{x})}$$

Since the value of $p^*(\beta|\mathbf{x})$ is find from equation (3.5)

$$p^*(\beta|\mathbf{x}) = \frac{\sum_{z_1=1}^{\infty} \sum_{z_2=1}^{\infty} \cdots \sum_{z_n=1}^{\infty} (1-p)^{\sum_{i=1}^{n} z_i} \beta^{n+\alpha-1} e^{-\beta\left(\sum_{i=1}^{n} z_i x_i + \frac{1}{\bar{\beta}}\right)}}{\sum_{z_1=1}^{\infty} \sum_{z_2=1}^{\infty} \cdots \sum_{z_n=1}^{\infty} \left\{ \frac{(1-p)^{\sum_{i=1}^{n} z_i}\Gamma(n+\alpha)}{\left(\sum_{i=1}^{n} z_i x_i + \frac{1}{\bar{\beta}}\right)^{n+\alpha}} \right\}}$$

(4.4)

Put the value of $p^*(\beta|\mathbf{x})$ from equation (3.5) in equation (4.4) we get

$$= \int_0^\infty \beta \frac{\sum_{z_{1=1}}^\infty \sum_{z_{2=1}}^\infty \cdots \sum_{z_{n=1}}^\infty (1-p)^{\sum_{i=1}^n z_i} \beta^{n+\alpha-1} e^{-\beta\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)}}{\sum_{z_{1=1}}^\infty \sum_{z_{2=1}}^\infty \cdots \sum_{z_{n=1}}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i} \Gamma(n+\alpha)}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}} d\beta$$

$$= (n+\alpha) \frac{\sum_{z_{1=1}}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha+1}} \right\}}{\sum_{z_{1=1}}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}}$$

(4.5)

which is the required result ∎

**Survival Function:**

Under the SELF the survival function for EL distribution is given by

$$S(t) = P(X > t)$$

$$= 1 - \left[ 1 - \frac{\ln\left(1 - (1-p)e^{-\beta t}\right)}{\ln p} \right]$$

$$= \frac{\ln\left[1 - (1-p)e^{-\beta t}\right]}{\ln p}$$

$$= \frac{1}{(-\ln p)} \sum_{j=1}^\infty \frac{(1-p)^j}{j} e^{-j\beta t}$$

(4.6)

Hence, under squared error loss function, the Bayes estimator of survival function is obtained as

$$\tilde{S}(t) = \int_0^\infty S(t) p^*(\beta|\boldsymbol{x}) d\beta$$

(4.7)

Put the value of $S(t)$ and $p^*(\beta|\boldsymbol{x})$ from equation (4.6) and equation (3.5) in equation (4.7) we get

$$= \int_0^\infty \frac{1}{(-\ln p)} \sum_{j=1}^\infty \frac{(1-p)^j}{j} e^{-j\beta t}$$

$$\times \frac{\sum_{z_{1=1}}^\infty \sum_{z_{2=1}}^\infty \cdots \sum_{z_{n=1}}^\infty (1-p)^{\sum_{i=1}^n z_i} \beta^{n+\alpha-1} e^{-\beta\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)}}{\sum_{z_{1=1}}^\infty \sum_{z_{2=1}}^\infty \cdots \sum_{z_{n=1}}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i} \Gamma(n+\alpha)}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}}$$

$$= \frac{\sum_{j=1}^\infty \sum_{z_{1=1}}^\infty \sum_{z_{2=1}}^\infty \cdots \sum_{z_{n=1}}^\infty \frac{(1-p)^j}{j} (1-p)^{\sum_{i=1}^\infty z_i} \times \int_0^\infty \beta^{n+\alpha-1} e^{-\beta\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta} + jt\right)} d\beta}{(-\ln p) \sum_{z_{1=1}}^\infty \sum_{z_{2=1}}^\infty \cdots \sum_{z_{n=1}}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i} \Gamma(n+\alpha)}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}}$$

$$= \frac{\sum_{j=1}^\infty \sum_{z_{1=1}}^\infty \sum_{z_{2=1}}^\infty \cdots \sum_{z_{n=1}}^\infty \left\{ \frac{\frac{(1-p)^j}{j} (1-p)^{\sum_{i=1}^\infty z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta} + jt\right)^{n+\alpha}} \right\}}{(-\ln p) \sum_{z_{1=1}}^\infty \sum_{z_{2=1}}^\infty \cdots \sum_{z_{n=1}}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}}$$

(4.8)

## V. ESTIMATION UNDER LINEX LOSS FUNCTION

Now we consider the estimation of parameter $\beta$ and survival function under the LINEX loss function. If we estimate parameter $\beta$ by, $\hat{\beta}$ the corresponding LINEX loss function is defined as

$$L(\hat{\beta}) = [e^{c\Delta} - c\Delta - 1], c \neq 0$$

(5.1)

where $\Delta = \hat{\beta} - \beta$. For $c > 0$, the loss function gives more weight age to overestimation where as for $c < 0$, underestimation gets more weightage. When $c \to 0$, the LINEX loss function approaches to

$$L(\hat{\beta}) = \frac{c^2(\hat{\beta} - \beta)^2}{2}$$

which is similar to the squared error loss function.

Under the LINEX loss function the Bayes estimator of $\beta$ is given by

$$\breve{\beta}_L = -\frac{1}{c} \ln E(e^{-c\beta})$$

(5.2)

where expectation is taken with respect to the posterior distribution of $\beta$. Now we have

$$E(e^{-c\beta}) = \int_0^\infty e^{-c\beta} p^*(\beta|\boldsymbol{x}) \, d\beta$$

(5.3)

From equation (3.5) putting the value of $p^*(\beta|\boldsymbol{x})$ in equation (5.3) we get

$$E(e^{-c\beta})$$

$$= \frac{\sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty (1-p)^{\sum_{i=1}^n z_i} \beta^{n+\alpha-1} e^{-\beta\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta} + c\right)}}{\sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i} \Gamma(n+\alpha)}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}}$$

(5.4)

$$= \frac{\sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta} + c\right)^{n+\alpha}} \right\}}{\sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}}$$

(5.5)

Using the above expression (5.5) in (5.2) leads to the Bayes estimator of $\beta$ as

$$\tilde{\beta}_L$$

$$= -\frac{1}{c} \ln \left\{ \frac{\sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta} + c\right)^{n+\alpha}} \right\}}{\sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}} \right\}$$

(5.6)

The Bayes risk of the estimator $\breve{\beta}_L$ under LINEX loss function is

$$\rho(\tilde{\beta}_L) = c(\tilde{\beta} - \tilde{\beta}_L).$$

For obtaining the Bayes estimator of survival function S(t) under the LINEX loss function, we observe that

$$e^{-cS(t)} = e^{\left[\ln[1-(1-p)e^{-\beta t}]\right]^{-\frac{c}{(\ln p)}}}$$

$$= \left[1 - (1-p)e^{-\beta t}\right]^{-\delta}$$

$$= \sum_{j=0}^\infty \frac{\Gamma(\delta+j-1)}{j! \, \Gamma(\delta-1)} (1-p)^j e^{-j\beta t}$$

where $\delta = \frac{c}{\ln p}$. Hence we observe that

$$E\left[e^{-cS(t)}\right]$$

$$= E\left[ \sum_{j=0}^\infty \frac{\Gamma(\delta+j-1)}{j! \, \Gamma(\delta-1)} (1-p)^j e^{-j\beta t} \right]$$

$$= \sum_{j=0}^\infty \frac{\Gamma(\delta+j-1)}{j! \, \Gamma(\delta-1)} (1-p)^j \sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty (1-p)^{\sum_{i=1}^n z_i}$$

$$\times \frac{\int_0^\infty \beta^{n+\alpha-1} e^{-\beta\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta} + j\beta t\right)} d\beta}{\Gamma(n+\alpha) \sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}}$$

$$= \frac{\sum_{j=0}^\infty \frac{\Gamma(\delta+j-1)}{j! \, \Gamma(\delta-1)} \sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \frac{(1-p)^{\sum_{i=1}^n z_i + j}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta} + jt\right)^{n+\alpha}}}{\sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}}$$

(5.7)

Hence the Bayes estimator of survival function under LINEX loss function is given by

$$\tilde{S}_L(t)$$

$$= -\frac{1}{c} \ln \left[ \frac{\sum_{j=0}^\infty \frac{\Gamma(\delta+j-1)}{j! \, \Gamma(\delta-1)} \sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \frac{(1-p)^{\sum_{i=1}^n z_i + j}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta} + jt\right)^{n+\alpha}}}{\sum_{z_1=1}^\infty \sum_{z_2=1}^\infty \cdots \sum_{z_n=1}^\infty \left\{ \frac{(1-p)^{\sum_{i=1}^n z_i}}{\left(\sum_{i=1}^n z_i x_i + \frac{1}{\beta}\right)^{n+\alpha}} \right\}} \right]$$

(5.8)

The risk associated with the Bayes estimator of survival function under LINEX loss function can be obtained as

    

$$\rho(\breve{\beta}_L) = c\left(\tilde{S}(t) - \tilde{S}_L(t)\right)$$

(5.9)

## VI. CONCLUDING REMARK:

CONCLUDING REMARKS The paper derives the posterior distribution and Bayes estimators assuming that the parameter p is known. However, if p is unknown, we may assume uniform or Beta prior distribution for p in interval $(0,1)$ and derive the conditional posterior distributions of ▯ given p and p given ▯ .Then the MCMC algorithm can be used to obtain the marginal distributions and Bayes estimators of parameters. The results of the paper can be easily extended when one considers type I censoring or type II censoring.

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